the gain of the amplifiers. The multiplier (a commercial type manufactured by G. A. Philbrick, Inc.) is followed by an amplifier and an RC integrator of variable integration time (22.5 to 90 seconds). The output of the integrator was fed into a recorder via a cathode follower to permit better registration of the time average of the signal.

 $R_0$  and  $R_2$  were carefully cooled to liquid helium temperatures and then their d-c. value was measured with a Wheatstone bridge ( $R_1$  and  $C_1$  were disconnected).  $R_1$  was set to approximately the value at which balance  $[\operatorname{Re}(\overline{v_0v_2^*}) = 0]$  was expected and  $C_1$  adjusted such that  $R_1C_1 \simeq R_2C_2$ . This was accomplished by connecting  $Z_1$  and  $Z_2$  in series and applying pulses across  $Z_1$  and  $Z_2$ .  $C_1$  was then adjusted until the shape of the pulses across the impedance  $Z_1+Z_2$  and  $Z_2$  were identical. This procedure adjusted the time constants  $\tau_1$  and  $\tau_2$  to approximately 10%. Similarly  $\tau_0$  and  $\tau_1$  were adjusted and the average setting of  $C_1$  was used when  $R_1$  was varied to achieve balance. If the change in  $R_1$  was large  $\tau_1$  had to be rebalanced and  $R_1$  reset. The temperature (room temperature) of  $R_1$  was read on an ordinary mercury thermometer placed on the outside of the resistance box and the value of  $R_1$  was read from the dial setting of the resistance box. The temperature of the helium bath was then calculated from equation (5).

The temperature of the helium bath was sometimes kept constant to better than 1 millidegree K by a temperature regulator (400 c.p.s.) similar to that of Boyle and Brown (1954). A stirrer was sometimes used to equalize the temperature, and the vapor pressure of the helium was measured on a mercury manometer with a cathetometer. A German silver tube (8-mm diameter) extending into the liquid surface was connected to the manometer. The temperature determined from equation (5) was then compared with the "1958 <sup>4</sup>He scale of temperatures" (Van Dijk and Durieux 1958; Brickwedde 1958).

## III. ERRORS AND LIMITATIONS OF THE THERMOMETER

# 1. Errors Which Can Be Represented by Noise-Current Sources in Shunt with the $\pi$ Network

These errors can be divided essentially into two groups: (a) errors due to the grid currents, (b) errors due to the finite input admittance.

(a) The grid current is made up of three parts; electrons arriving at the grid  $(I_1)$ , electrons emitted from the grid by photoelectric emissions  $(I_2)$ , and positive ions arriving at the grid  $(I_3)$ . All three currents are independent of each other. The grid of the triode acts like the anode of a diode; for  $I_1$  it acts like the anode in the exponential part of its characteristic and for  $I_2$  and  $I_3$  it acts like the anode of a saturated diode. Therefore the shot noise due to the grid current is:

(6) 
$$i_s^2 = 2e(I_1 + I_2 + I_3) df.$$

0

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The net grid current is  $I_g = I_1 - I_2 - I_3$  and thus  $i_s^2 \ge 2eI_g df$ .

(b) The real part of the dynamic input admittance which is a function of frequency consists of three components: the ohmic loss in the input circuit, the cold loss of the first stage (leakage around the bulb of the tube, losses in

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the socket, etc.), and part of the load of the first stage which is reflected into the input due to the grid-to-plate capacity. In this experiment a cascode input was used which has the advantage that the grounded grid stage reduces the capacitive feedback from output to input without introducing partition noise. However, due to the finite feedback shot noise of the grounded grid stage contributes also to this error.

The errors in (a) and (b) can be represented by noise-current sources in shunt with the  $\pi$  network. The noise temperature which must be ascribed to this input conductance cannot be determined by calculation because its components and their noisiness are not readily estimated. If one assumes that the real part of the input impedance to the amplifiers is  $R_g$  and its effective temperature is  $T_g = \alpha T_1$ , where  $\alpha$  is a constant and  $T_1$  room temperature, and if one also assumes that both amplifiers have the same input characteristics and that  $R_g \gg R_0$  and  $R_0 \simeq R_2$ , then the error in the absolute temperature due to shunt current sources is:

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(7) 
$$\epsilon = \frac{\Delta T}{T} \simeq \frac{R_0}{2kT} \left[ e(I_1 + I_2 + I_3) + 2k \frac{\alpha T_1}{R_g} \right] = \frac{A}{T},$$

where A is a constant for one particular thermometer.

### 2. Errors Due to Current Flow in the Resistors

Nyquist's law is based upon the assumption that the circuit is a passive network. This requires that no currents are flowing through the resistors  $R_0$ ,  $R_1$ , and  $R_2$ . To minimize thermoelectric effects dissimilar materials between the amplifiers and resistors in the network were avoided and the voltage due to this effect was measured to be smaller than 3  $\mu v$  at the inputs of the amplifiers when  $R_0$  and  $R_2$  were at helium temperatures. The grid current  $I_1 - (I_2 + I_3)$  was approximately  $2.6 \times 10^{-9}$  amperes. Because thin metal layer resistors  $(R_0 \text{ and } R_2)$  consist of a large number of very small conducting particles in loose contact, contact noise may be generated if a current is passed through the resistors. Christenson and Pearson (1936) did not find any contact noise in thin solid carbon filaments when large currents were passed through the specimen. Also Bittel and Scheidhauer (1956) found no noise in addition to the thermal noise when a current was passed through metallic conductors between 45 c.p.s. and 11.5 kc/s. Therefore thin solid metal layer resistors should be free of any noise in excess of thermal noise and Nyquist's law should hold accurately for the above small currents.

## 3. Non-linearities and Amplifier Noise

Non-linearities in the amplifiers, the multiplier, and the integrator are another source of error. Due to non-linearities the recorder deflection will be increased by an increment proportional to  $(|\overline{v_0^2}|\overline{v_2^2}|)^{\frac{1}{2}}$ . Because for  $Z_1 = \infty$  as well as for balance (see equation 5) the correlation coefficient should be zero for no distortion of the signal, the variance should be the same for both cases. Because both amplifiers were built on different chassis and shielded from each other, the coupling capacity between the amplifiers must have been very small. The zero for balance of the recorder was then determined by grounding

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